CHAPTER 4

MOTION

Objectives

The major goals of this chapter are to enable you to:

1. Distinguish between speed and velocity.
2. Use vectors to illustrate and solve velocity problems.
3. Distinguish between velocity and acceleration.
4. Utilize vectors to illustrate and solve acceleration problems.
5. Analyze the motion of an object in free fall.
6. Solve two-dimensional motion problems.
7. Calculate the range of projectile motion.

Motion is a change of position. Velocity and acceleration describe important kinds of motion. An analysis of motion helps introduce the real nature of physics—to understand the nature and behavior of the physical world.

4.1 Speed Versus Velocity

Motion is defined as an object's change in position. One of the most frequently used descriptors of motion is speed.

Describing and measuring an object's motion is critical in many aspects of science, engineering, and everyday life. In baseball, the speed of a fastball is measured to determine the prowess of a pitcher. Medical technicians use Doppler radar devices to measure the speed of blood passing through arteries. Automotive engineers synchronize the motion of pistons, valves, and drive shafts to ensure that an engine operates efficiently. Finally, automobile drivers frequently refer to their speedometers to gauge how fast they are traveling.

Figure 4.1 A speedometer measures speed but not velocity.

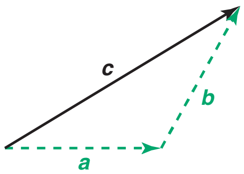
Speed, as measured on a speedometer, is the distance traveled per unit of time. The speed of an automobile is represented in either miles per hour or kilometres per hour (Fig. 4.1). These units actually help define the formula for calculating speed:

$$\text{speed} = \frac{\text{distance traveled}}{\text{time to move that distance}}$$

Speed is a scalar value, for it shows only the magnitude of the position change per unit of time and does not indicate a direction. The unit for speed is a distance unit divided by a time unit, such as miles per hour (mi/h), kilometres per hour (km/h), metres per second (m/s), and feet per second (ft/s). For example, if you drive $35\bar{0}$ mi in 7.00 h, your average speed is

$$\frac{35\bar{0}\text{ mi}}{7.00\text{ h}} = 50.0\text{ mi/h}$$

Figure 4.2 A person walks along path *a* and then turns to follow path *b*. The distance traveled is the total distance of *a* and *b*. The displacement is represented by the distance of *c*.



Speed represents how fast something is moving, yet it does not indicate the direction in which it is traveling. Suppose you started driving from Chicago at a speed of 50 mi/h for 6 h. Where did you end your trip? You may have driven 50 mi/h southwest toward St. Louis, 50 mi/h northeast toward Detroit, 50 mi/h southeast toward Louisville, or 50 mi/h in a loop that brought you back to Chicago. Although speed may indicate how fast you are moving, it may not give you all the information you need to solve a problem.

Distance traveled must be distinguished from displacement. Whereas *distance* traveled may follow a path that is not straight, *displacement* is the net change of position of an object. It is represented by a straight line from the initial position to the final position and is a vector because it has both magnitude and direction (Fig. 4.2).

The **velocity** of an object is the rate of motion in a particular direction. Velocity is a vector that not only represents the speed, but also indicates the direction of motion. The relationship may be expressed by the equation

$$v_{\text{avg}} = \frac{s}{t}$$

or

$$\boxed{s = v_{\text{avg}}t}$$

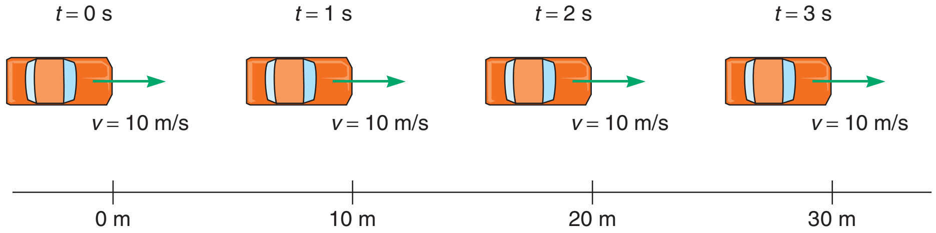
where s = displacement
v_{avg} = average velocity
t = time

This equation is used to find either average speed (a scalar quantity) or the magnitude of the velocity (a vector quantity). Remember that if indicating velocity, the direction must

be included with the speed. Therefore, a speed of 50 mi/h would be written 50 mi/h northeast, 50 mi/h up, or 50 mi/h 30° east of south as a velocity.

Figure 4.3 shows an illustration of a car traveling at a constant velocity of 10 m/s to the right. Note that its displacement is 10 m to the right during each second of travel.

Figure 4.3 The velocity, displacement, and time for a car traveling at a constant velocity of 10 m/s to the right is shown in 1-s intervals.



EXAMPLE 1

Find the average speed of an automobile that travels 160 km in 2.0 h.

Data:

$$s = 160 \text{ km}$$
$$t = 2.0 \text{ h}$$
$$v_{avg} = ?$$

Basic Equation:

$$s = v_{avg}t$$

Working Equation:

$$v_{avg} = \frac{s}{t}$$

Substitution:

$$v_{avg} = \frac{160 \text{ km}}{2.0 \text{ h}} = 8\bar{0} \text{ km/h}$$

EXAMPLE 2

An airplane flies $35\bar{0}0$ mi in 5.00 h. Find its average speed.

Data:

$$s = 35\bar{0}0 \text{ mi}$$
$$t = 5.00 \text{ h}$$
$$v_{avg} = ?$$

Basic Equation:

$$s = v_{avg}t$$

Working Equation:

$$v_{avg} = \frac{s}{t}$$

Substitution:

$$v_{avg} = \frac{350\bar{0} \text{ mi}}{5.00 \text{ h}}$$
$$= 70\bar{0} \text{ mi/h}$$

EXAMPLE 3

Find the velocity of a plane that travels $60\bar{0}$ km due north in 3 h 15 min.

Data:

$$s = 60\bar{0} \text{ km}$$
$$t = 3 \text{ h } 15 \text{ min} = 3.25 \text{ h}$$
$$v_{avg} = ?$$

Basic Equation:

$$s = v_{avg}t$$

Working Equation:

$$v_{avg} = \frac{s}{t}$$

Substitution:

$$v_{avg} = \frac{60\bar{0} \text{ km}}{3.25 \text{ h}}$$
$$= 185 \text{ km/h}$$

The direction is north. Thus, the velocity is 185 km/h due north.

Until now, our study of velocity has assumed a fixed observation point. The frame of reference can also be important for determining relative velocity. Paddling a canoe into a headwind may produce a net velocity of zero for the paddler. Another example is the flight of an airplane in which a crosswind, or in fact wind from any direction, will affect the airplane's velocity with respect to the ground. The airplane's final velocity is calculated by taking into account the velocity of the airplane in calm air and the velocity of any wind that the airplane encounters.

To find the sum (resultant vector) of velocity vectors, use the component method as outlined in Chapter 3.

EXAMPLE 4

A plane is flying due north (at 90°) at 265 km/h and encounters a wind from the east (at 180°) at 55.0 km/h. What is the plane's new velocity with respect to the ground in standard position? Assume that the plane's new velocity is the vector sum of the plane's original velocity and the wind velocity.

First, graph the plane's old velocity as the y-component and the wind velocity as the x-component (Fig. 4.4). The resultant vector is the plane's new velocity with respect to the ground. Find angle α as follows:

$$\tan \alpha = \frac{\text{side opposite } \alpha}{\text{side adjacent to } \alpha}$$

$$\tan \alpha = \frac{265 \text{ km/h}}{55.0 \text{ km/h}} = 4.818$$

$$\alpha = 78.3°$$

then

$$\theta = 180° - 78.3° = 101.7°$$

Find the magnitude of the new velocity (ground speed) using the Pythagorean theorem:

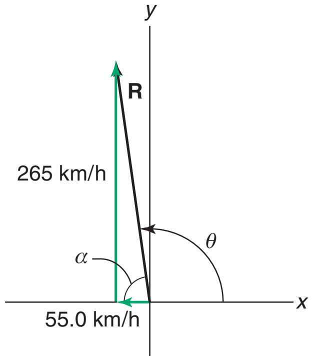
$$|\mathbf{R}| = \sqrt{|\mathbf{R}_x|^2 + |\mathbf{R}_y|^2}$$

$$|\mathbf{R}| = \sqrt{(55.0 \text{ km/h})^2 + (265 \text{ km/h})^2}$$

$$= 271 \text{ km/h}$$

That is, the new velocity of the plane is 271 km/h at 101.7°.

Figure 4.4

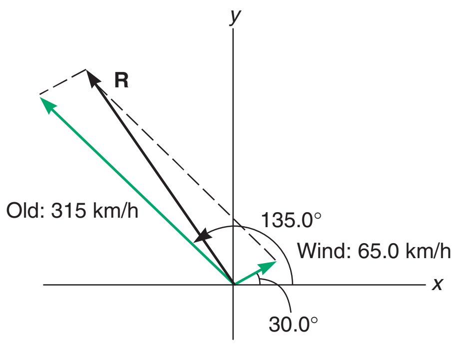


EXAMPLE 5

A plane is flying northwest (at 135.0°) at 315 km/h and encounters a wind from 30.0° south of west (at 30.0°) at 65.0 km/h. What is the plane's new velocity with respect to the ground in standard position? Assume that the plane's new velocity is the vector sum of the plane's original velocity and the wind velocity.

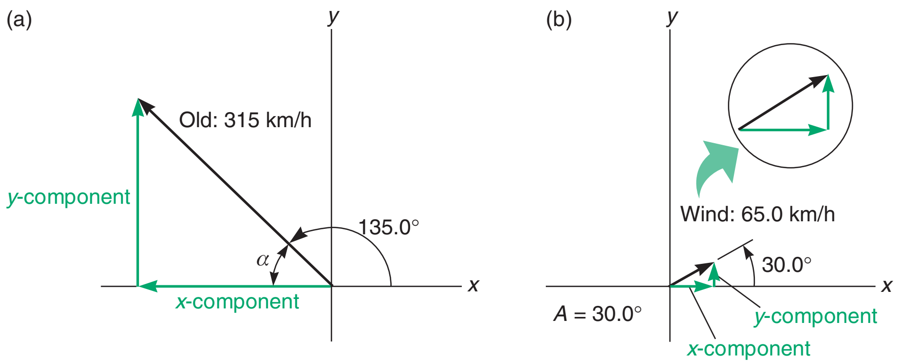
First, graph the plane's old velocity and the wind velocity as vectors in standard position (Fig. 4.5). The resultant vector is the plane's new velocity with respect to the ground.

Figure 4.5



Then, find the x- and y-components of the plane's old velocity and the wind velocity using Fig. 4.6.

Figure 4.6



Plane: See Fig. 4.6(a). $\alpha = 180° - 135.0° = 45.0°$

x-component	y-component
$\cos\alpha = \dfrac{\text{side adjacent to } \alpha}{\text{hypotenuse}}$	$\sin\alpha = \dfrac{\text{side opposite } \alpha}{\text{hypotenuse}}$
$\cos 45.0° = \dfrac{x\text{-component}}{315 \text{ km/h}}$	$\sin 45.0° = \dfrac{y\text{-component}}{315 \text{ km/h}}$
$(315 \text{ km/h})(\cos 45.0°) = x\text{-component}$	$(315 \text{ km/h})(\sin 45.0°) = y\text{-component}$
$223 \text{ km/h} = x\text{-component}$	$223 \text{ km/h} = y\text{-component}$
Thus, $x\text{-component} = -223 \text{ km/h}$	$y\text{-component} = +223 \text{ km/h}$

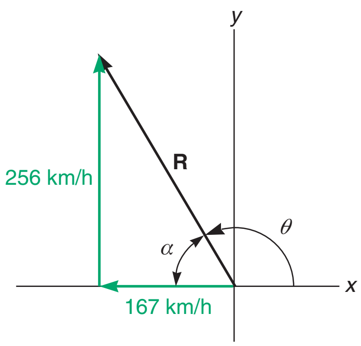
Wind: See Fig. 4.6(b).

x-component	y-component
$\cos\alpha = \dfrac{\text{side adjacent to } \alpha}{\text{hypotenuse}}$	$\sin\alpha = \dfrac{\text{side opposite } \alpha}{\text{hypotenuse}}$
$\cos 30.0° = \dfrac{x\text{-component}}{65.0 \text{ km/h}}$	$\sin 30.0° = \dfrac{y\text{-component}}{65.0 \text{ km/h}}$
$(65.0 \text{ km/h})(\cos 30.0°) = x\text{-component}$	$(65.0 \text{ km/h})(\sin 30.0°) = y\text{-component}$
$56.3 \text{ km/h} = x\text{-component}$	$32.5 \text{ km/h} = y\text{-component}$
Thus, $x\text{-component} = +56.3 \text{ km/h}$	$y\text{-component} = +32.5 \text{ km/h}$

To find **R:**

	x-component	y-component	
Plane:	-223 km/h	$+223$ km/h	
Wind:	$+56.3$ km/h	$+32.5$ km/h	
Sum:	-167 km/h	$+256$ km/h	(Round each component sum to its least precise component.)

Figure 4.7



Find angle α from Fig. 4.7 as follows:

$$\tan\alpha = \frac{\text{side opposite } \alpha}{\text{side adjacent to } \alpha}$$

$$\tan\alpha = \frac{256 \text{ km/h}}{167 \text{ km/h}} = 1.533$$

$$\alpha = 56.9°$$

and

$$\theta = 180° - 56.9° = 123.1°$$

Find the magnitude of **R** using the Pythagorean theorem:

$$|\mathbf{R}| = \sqrt{|\mathbf{R}_x|^2 + |\mathbf{R}_y|^2}$$

$$|\mathbf{R}| = \sqrt{(167\text{ km/h})^2 + (256\text{ km/h})^2}$$

$$= 306\text{ km/h}$$

That is, the new velocity of the plane is 306 km/h at 123.1°.

PHYSICS CONNECTIONS

Vectors Across Rivers

Crossing the Hudson River, which separates New York City from New Jersey, can be challenging. A working knowledge of velocity and vectors is absolutely essential, especially when attempting to cross the river in strong currents, brisk winds, driving rain, and dense fog. In addition, maneuvering between barges, cruise ships, recreational boaters, and driftwood can make the job even more difficult.

Ferry captains like Mike and John combine vectors every time they cross the river. Captain Mike said, "At times crossing the river can be quite tricky. Your ferry might be pointing directly across the river, but the current is pushing you farther down river. In order to combat this, you need to change the boat's heading so when your velocity and the current's velocity combine, you arrive at your planned destination" (Fig. 4.8).

Sometimes, when the current and wind are strong and headed in the same direction, ferryboats can appear to be heading up to 45° away from their destination, yet still travel directly across the river. In such situations, docking can be nerve-racking (Fig. 4.9).

Figure 4.8 An example of how the velocity of a boat and the velocity of the current are combined so the resultant velocity is directed toward the desired location.

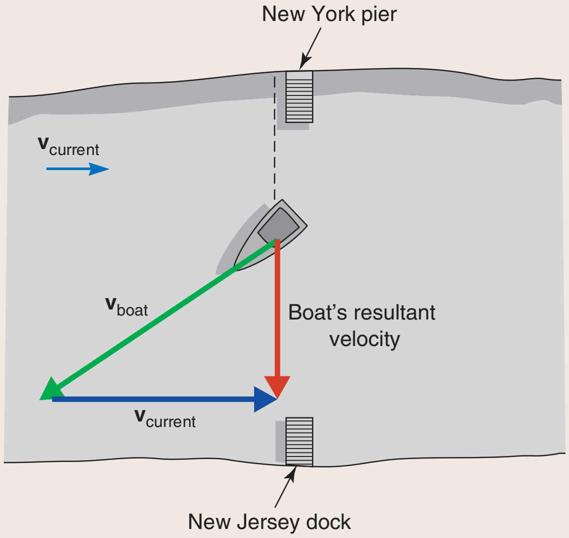
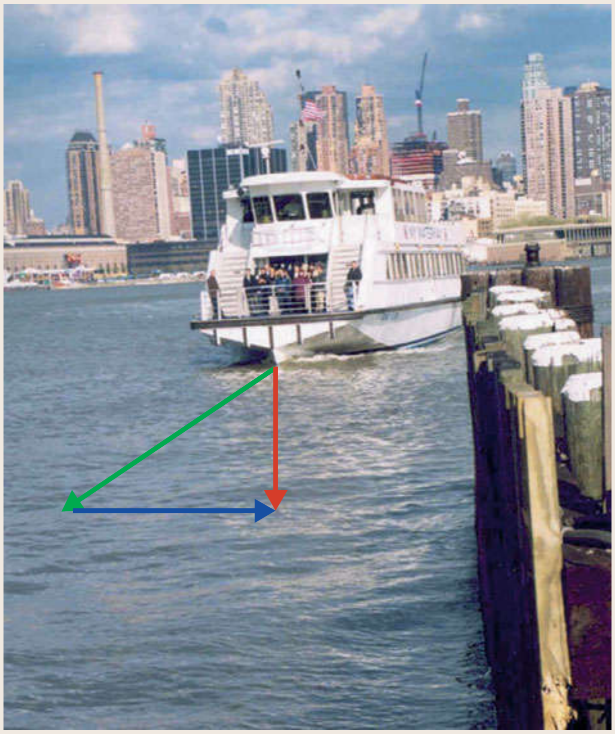


Figure 4.9 Although the boat is not pointed toward the dock, the combination of the boat's velocity (green vector) plus the current's velocity (blue vector) results in a perfect docking (red vector).



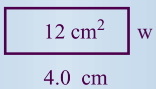
The two experienced sea captains say that vectors are even more important in the open seas. Captain Mike said, "Combining your boat's velocity vector with the current and wind vectors can mean the difference between arriving at your home port or at a port 100 miles away." These days the process is easier with the use of radar and computer navigation equipment and software. Looking at the monitor, Captain Mike said, "We can see the velocity of the current and our boat's intended velocity directly on the monitor. Instead of our combining the vectors, the computer can instantaneously combine them and provide the resultant velocity" (Fig. 4.10).

Figure 4.10 (a) Captain Mike combines velocity vectors every time he navigates across the Hudson River.

Figure 4.10 (b) Captain John's high-speed ferry is equipped with computers that automatically combine and display the boat's resultant velocity vector.

PROBLEM SOLVING

SKETCH



DATA

$A = 12\text{ cm}^2$, $l = 4.0\text{ cm}$, $w = ?$

BASIC EQUATION

$A = lw$

WORKING EQUATION

$$w = \frac{A}{l}$$

SUBSTITUTION

$$w = \frac{12\text{ cm}^2}{4.0\text{ cm}} = 3.0\text{ cm}$$

PROBLEMS 4.1

Find the average speed (in the given units) of an auto that travels each distance in the given time.

1. 150 mi in 3.0 h (in mi/h)
2. 190 m in 8.5 s (in m/s)
3. 8550 m in 6 min 35 s (in m/s)
4. 45 km in 0.50 h (in km/h)
5. 785 ft in 11.5 s (in ft/s)
6. Find the average speed (in mi/h) of a racing car that turns a lap on a 1.00-mi oval track in 30.0 s.
7. While driving at $9\overline{0}$ km/h, how far can you travel in 3.5 h?
8. While driving at $9\overline{0}$ km/h, how far (in metres) do you travel in 1.0 s?
9. An automobile is traveling at 55 mi/h. Find its speed
   (a) in ft/s. (b) in m/s. (c) in km/h.
10. An automobile is traveling at 22.0 m/s. Find its speed
    (a) in km/h. (b) in mi/h. (c) in ft/s.
11. A semi-trailer truck traveling $10\overline{0}$ km/h continues for 2.75 h. How far does it go?
12. A flatbed truck travels for 3.85 h at 105 km/h. How far does it go?
13. The average speed of a garbage truck is 60.0 km/h. How long does it take for the truck to travel 265 km?
14. A highway maintenance truck has an average speed of 55.0 km/h. How far does it travel in 3.65 h?

Find the velocity for each displacement and time.

15. 160 km east in 2.0 h
16. 1$\bar{0}$0 km north in 3.0 h
17. 10$\bar{0}$0 mi south in 8.00 h
18. 31.0 mi west in 0.500 h
19. 275 km at 3$\bar{0}$° south of east in 4.50 h
20. 426 km at 45° north of west in 2.75 h
21. Milwaukee is 121 mi (air miles) due west of Grand Rapids. Maria drives 255 mi in 4.75 h from Grand Rapids to Milwaukee around Lake Michigan. Find (a) her average driving speed and (b) her average travel velocity.
22. Telluride, Colorado, is 45 air miles at 11° east of north of Durango. On a winter day, Chuck drove 120 mi from Durango to Telluride around a mountain in $4\frac{1}{4}$ h including a traffic delay. Find (a) his average driving speed and (b) his average travel velocity.

In Problems 23–30, assume that the plane's new velocity is the vector sum of the plane's original velocity and the wind velocity.

23. A plane is flying due north at 325 km/h and encounters a wind from the south at 45 km/h. What is the plane's new velocity with respect to the ground in standard position?
24. A plane is flying due west at 275 km/h and encounters a wind from the west at 8$\bar{0}$ km/h. What is the plane's new velocity with respect to the ground in standard position?
25. A plane is flying due west at 235 km/h and encounters a wind from the north at 45.0 km/h. What is the plane's new velocity with respect to the ground in standard position?
26. A plane is flying due north at 185 mi/h and encounters a wind from the west at 35.0 mi/h. What is the plane's new velocity with respect to the ground in standard position?
27. A plane is flying southwest at 155 mi/h and encounters a wind from the west at 45.0 mi/h. What is the plane's new velocity with respect to the ground in standard position?
28. A plane is flying southeast at 215 km/h and encounters a wind from the north at 75.0 km/h. What is the plane's new velocity with respect to the ground in standard position?
29. A plane is flying at 25.0° north of west at 19$\bar{0}$ km/h and encounters a wind from 15.0° north of east at 45.0 km/h. What is the plane's new velocity with respect to the ground in standard position?
30. A plane is flying at 36.0° south of west at 15$\bar{0}$ mi/h and encounters a wind from 75.0° north of east at 55.0 mi/h. What is the plane's new velocity with respect to the ground in standard position?

4.2 Acceleration

When shopping for a new vehicle, most consumers choose to ignore its maximum speed but instead pay particular attention to how quickly the vehicle can change its speed from 0 mi/h to 60 mi/h. From the new-car-buyer's perspective, the less time the car takes to achieve a speed of 60 mi/h, the greater will be its acceleration and the easier it will be to get onto a highway or pass a slow-moving vehicle.

Acceleration *is the change in velocity per unit time.* In other words, acceleration measures how quickly velocity changes. That is,

$$\begin{aligned}\text{average acceleration} &= \frac{\text{change in velocity (or speed)}}{\text{elapsed time}} \\ &= \frac{\text{final velocity} - \text{initial velocity}}{\text{time}}\end{aligned}$$

This relationship can be expressed by the equation

$$a = \frac{\Delta v}{t} = \frac{v_f - v_i}{t}$$

or

$$\Delta v = at$$

where Δv = change in velocity (or speed)
a = acceleration
t = time

The Greek letter Δ (capital delta) is used to mean "change in."

EXAMPLE 1

A dragster starts from rest (velocity = 0 ft/s) and attains a speed of $15\bar{0}$ ft/s in 10.0 s. Find its acceleration.

Data:

$$\Delta v = 15\bar{0}\text{ ft/s} - 0\text{ ft/s} = 15\bar{0}\text{ ft/s}$$
$$t = 10.0\text{ s}$$
$$a = ?$$

Basic Equation:

$$\Delta v = at$$

Working Equation:

$$a = \frac{\Delta v}{t}$$

Substitution:

$$a = \frac{15\bar{0}\text{ ft/s}}{10.0\text{ s}}$$
$$= 15.0\,\frac{\text{ft/s}}{\text{s}}\text{ or 15.0 feet per second per second}$$

Recall from arithmetic that to simplify fractions in the form

$$\frac{\frac{a}{b}}{\frac{c}{d}}$$

we divide by the denominator; that is, invert and multiply:

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \div \frac{c}{d} = \frac{a}{b}\cdot\frac{d}{c} = \frac{ad}{bc}$$

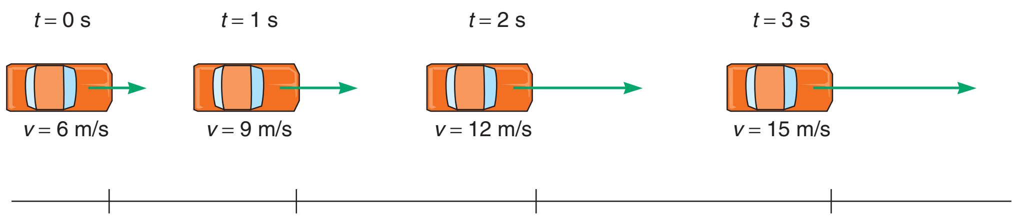
Use this idea to simplify the units 15.0 feet per second per second:

$$\frac{\frac{15.0\text{ ft}}{\text{s}}}{\frac{\text{s}}{1}} = \frac{15.0\text{ ft}}{\text{s}} \div \frac{\text{s}}{1} = \frac{15.0\text{ ft}}{\text{s}}\cdot\frac{1}{\text{s}} = \frac{15.0\text{ ft}}{\text{s}^2}\text{ or 15.0 ft/s}^2$$

The units of acceleration are usually ft/s^2 or m/s^2.

When the speed of an automobile increases from rest to 5 mi/h in the first second to 10 mi/h in the next second and to 15 mi/h in the third second, its acceleration is $5\,\dfrac{\text{mi/h}}{\text{s}}$. That is, its increase in speed is 5 mi/h during each second. In Fig. 4.11 an automobile increases in speed from 6 m/s to 9 m/s in the first second, to 12 m/s in the next second, and to 15 m/s in the third second, so its acceleration is $3\,\dfrac{\text{m/s}}{\text{s}}$, usually written 3 m/s^2. This means that the speed of the automobile increases 3 m/s during each second.

Figure 4.11 This car is speeding up with a constant acceleration. Note how the distance covered and the velocity change during each time interval.



EXAMPLE 2

A car accelerates from 45 km/h to $8\bar{0}$ km/h in 3.00 s. Find its acceleration (in m/s^2).

Data:

$$\Delta v = 8\bar{0}\text{ km/h} - 45\text{ km/h} = 35\text{ km/h}$$
$$t = 3.00\text{ s}$$
$$a = ?$$

Basic Equation:

$$\Delta v = at$$

Working Equation:

$$a = \frac{\Delta v}{t}$$

Substitution:

$$a = \frac{35\ \cancel{\text{km}}/\cancel{\text{h}}}{3.00\text{ s}} \times \frac{1000\text{ m}}{1\ \cancel{\text{km}}} \times \frac{1\ \cancel{\text{h}}}{3600\text{ s}}$$
$$= 3.2\text{ m/s}^2$$

Note the use of the conversion factors to change the units km/h/s to m/s^2.

EXAMPLE 3

A plane accelerates at 8.5 m/s^2 for 4.5 s. Find its increase in speed (in m/s).

Data:

$$a = 8.5\text{ m/s}^2$$
$$t = 4.5\text{ s}$$
$$\Delta v = ?$$

Basic Equation:

$$\Delta v = at$$

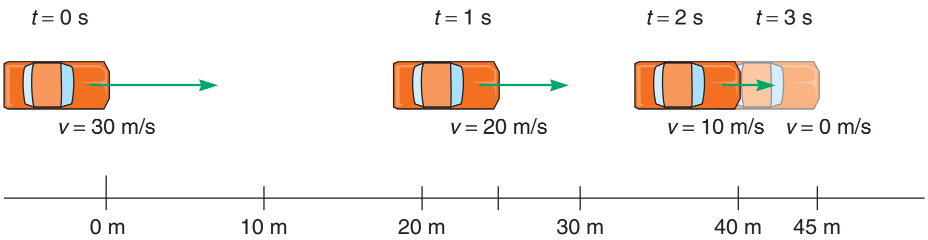
Working Equation: Same

Substitution:

$$\begin{aligned} \Delta v &= (8.5\ \text{m/s}^2)(4.5\ \text{s}) \\ &= 38\ \text{m/s} \end{aligned} \qquad \boxed{\frac{\text{m}}{\text{s}^2} \times \text{s} = \frac{\text{m}}{\text{s}}}$$

Acceleration means more than just an increase in speed. In fact, since velocity is the speed of an object and its direction of motion, acceleration can mean speeding up, slowing down, or changing direction. The next example illustrates negative acceleration (sometimes called deceleration). **Deceleration** is an acceleration that usually indicates that an object is slowing down (Fig. 4.12). Acceleration when an object changes direction will be discussed in Chapter 9.

Figure 4.12 This car is slowing down with a constant acceleration of $-10\ \text{m/s}^2$. Note how the distance covered and the velocity change during each unit of time interval.



Sometimes the direction of the velocity or the acceleration of an object is understood and does not need to be stated explicitly. For example, if a car is accelerating on a straight road, both the direction of the velocity and the direction of the acceleration are along the road in the direction in which the car is moving. When the car begins to slow down, the direction of the velocity is understood to be along the road in the direction pointing ahead of the car, and the direction of its acceleration (deceleration) is understood to be along the road in the direction pointing behind the car.

EXAMPLE 4

A driver steps off the gas pedal and coasts at a rate of $-3.00\ \text{m/s}^2$ for 5.00 s. Find the driver's new speed if she was originally traveling at a velocity of 20.0 m/s. (The negative acceleration indicates that the acceleration is in the opposite direction of the velocity; that is, the object is slowing down.)

Data:

$$\begin{aligned} a &= -3.00\ \text{m/s}^2 \\ t &= 5.00\ \text{s} \\ v_i &= 20.0\ \text{m/s} \end{aligned}$$

Basic Equation:

$$\begin{aligned} \Delta v &= at \\ v_f - v_i &= at \end{aligned}$$

Working Equation:

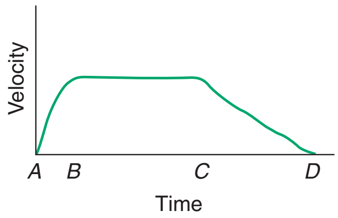
$$v_f = v_i + at$$

Substitution:

$$v_f = 20.0 \text{ m/s} + (-3.00 \text{ m/s}^2)(5.00 \text{ s})$$
$$= 5.0 \text{ m/s}$$

Acceleration has both magnitude and direction. Consider a high-speed train moving out of the station due east (positive direction). Its velocity increases in magnitude until it reaches its cruising speed (Fig. 4.13). Its acceleration is greatest at the start (from *A* to *B*), when its increase in speed is largest. When the train is moving at relatively constant velocity (from *B* to *C*), its acceleration is near zero. Then as the train approaches the next station (from *C* to *D*), its speed decreases as it decelerates, thus creating a negative acceleration.

Figure 4.13 Motion of a high-speed train going from one station to another. When the speed increases, the acceleration is positive. When the speed is constant, the acceleration is zero. When the speed decreases, the acceleration is negative.



PROBLEMS 4.2

An automobile changes speed as shown. Find its acceleration.

	Speed Change	*Time Interval*	*Find a*
1.	From 0 to 15 m/s	1.0 s	in m/s^2
2.	From 0 to 18 m/s	3.0 s	in m/s^2
3.	From $6\overline{0}$ ft/s to $7\overline{0}$ ft/s	1.0 s	in ft/s^2
4.	From 45 m/s to 65 m/s	2.0 s	in m/s^2
5.	From 25 km/h to $9\overline{0}$ km/h	5.6 s	in m/s^2
6.	From $1\overline{0}$ mi/h to $5\overline{0}$ mi/h	3.5 s	in ft/s^2

7. A dragster starts from rest and reaches a speed of 62.5 m/s in 10.0 s. Find its acceleration (in m/s^2).
8. A car accelerates from 25 mi/h to 55 mi/h in 4.5 s. Find its acceleration (in ft/s^2).
9. A train accelerates from $1\overline{0}$ km/h to $11\overline{0}$ km/h in 2 min 15 s. Find its acceleration (in m/s^2).

A plane accelerates at 30.0 ft/s^2 for 3.30 s. Find its increase in speed

10. in ft/s. 11. in mi/h.

A rocket accelerates at 10.0 m/s^2 from rest for 20.0 s. Find its increase in speed

12. in m/s. 13. in km/h.
14. How long (in seconds) does it take for a rocket sled accelerating at 15.0 m/s^2 to change its speed from 20.0 m/s to 65.0 m/s?
15. What is the acceleration of a road grader that goes from rest to 10.0 km/h in 5.20 s?
16. What is the acceleration of a compactor that goes from rest to 20.0 km/h in 4.80 s?
17. How long (in seconds) does it take for a truck accelerating at 1.50 m/s^2 to go from rest to 90.0 km/h?
18. How long (in seconds) does it take for a car accelerating at 3.50 m/s^2 to go from rest to $12\overline{0}$ km/h?
19. A bulldozer accelerates from rest to 3.00 m/s in 4.20 s. What is its acceleration?
20. A pickup truck pulling a trailer accelerates at 3.25 m/s^2 for 5.80 s. If it starts from rest, what is its final velocity?
21. The speed of a delivery van increases from 2.00 m/s at 1.00 s to 16.0 m/s at 4.50 s. What is its average speed?
22. A go-cart rolls backward down a driveway. We define forward speed as positive and backward speed as negative. The cart's speed changes from −2.00 m/s to −9.00 m/s in 2.00 s. What is its acceleration?
23. A stock car is moving at 25.0 m/s when the driver applies the brakes. If it stops in 3.00 s, what is its average acceleration?
24. If the car in Problem 23 took twice as long to stop, what would its acceleration be?

PROBLEM SOLVING

SKETCH

12 cm^2 w

4.0 cm

DATA

$A = 12 \text{ cm}^2$, $l = 4.0$ cm, w = ?

BASIC EQUATION

$A = lw$

WORKING EQUATION

$$w = \frac{A}{l}$$

SUBSTITUTION

$$w = \frac{12 \text{ cm}^2}{4.0 \text{ cm}} = 3.0 \text{ cm}$$

25. If the car in Problem 23 was going twice as fast but was able to stop in the same time, what would its acceleration be?
26. If the car in Problem 23 was going twice the speed and stopped in twice the time, what would its acceleration be?

4.3 Uniformly Accelerated Motion and Free Fall

Uniformly accelerated motion of an object occurs when its acceleration is constant; examples are a ball rolling down a straight incline, a car increasing its speed at a constant rate, and a ball dropped from a building. The following equations apply to uniformly accelerated motion and freely falling bodies. Some of the derivations are beyond the scope of this text.

ACCELERATED MOTION

1. $v_{avg} = \dfrac{v_f + v_i}{2}$
2. $a = \dfrac{v_f - v_i}{t}$
3. $v_f = v_i + at$
4. $s = v_i t + \frac{1}{2}at^2$
5. $s = \frac{1}{2}(v_f + v_i)t$
6. $2as = v_f^2 - v_i^2$

where s = displacement, v_f = final velocity, v_i = initial velocity, v_{avg} = average velocity, a = constant acceleration, t = time

Now consider some problems using these equations, applying our problem-solving method.

EXAMPLE 1

The average velocity of a rolling freight car is 2.00 m/s. How long does it take for the car to roll 15.0 m?

Data:

$$s = 15.0 \text{ m}$$
$$v_{avg} = 2.00 \text{ m/s}$$
$$t = ?$$

Basic Equation:

$$s = v_{avg}t$$

Working Equation:

$$t = \frac{s}{v_{avg}}$$

Substitution:

$$t = \frac{15.0 \text{ m}}{2.00 \text{ m/s}} = 7.50 \text{ s}$$

$$\boxed{\frac{\text{m}}{\text{m/s}} = \text{m} \div \frac{\text{m}}{\text{s}} = \text{m} \cdot \frac{\text{s}}{\text{m}} = \text{s}}$$

EXAMPLE 2

A dragster starting from rest reaches a final velocity of 318 km/h. Find its average velocity.

Data:

$$\begin{aligned} v_i &= 0 \\ v_f &= 318 \text{ km/h} \\ v_{\text{avg}} &= ? \end{aligned}$$

Basic Equation:

$$v_{\text{avg}} = \frac{v_f + v_i}{2}$$

Working Equation: Same

Substitution:

$$\begin{aligned} v_{\text{avg}} &= \frac{318 \text{ km/h} + 0 \text{ km/h}}{2} \\ &= 159 \text{ km/h} \end{aligned}$$

EXAMPLE 3

A train slowing to a stop has an average acceleration of -3.00 m/s^2. [Note that a minus $(-)$ acceleration is commonly called *deceleration*, meaning that the train is slowing down.] If its initial velocity is 30.0 m/s, how far does it travel in 4.00 s?

Data:

$$\begin{aligned} a &= -3.00 \text{ m/s}^2 \\ v_i &= 30.0 \text{ m/s} \\ t &= 4.00 \text{ s} \\ s &= ? \end{aligned}$$

Basic Equation:

$$s = v_i t + \tfrac{1}{2}at^2$$

Working Equation: Same

Substitution:

$$\begin{aligned} s &= (30.0 \text{ m/s})(4.00 \text{ s}) + \tfrac{1}{2}(-3.00 \text{ m/s}^2)(4.00 \text{ s})^2 \\ &= 12\bar{0} \text{ m} - 24.0 \text{ m} \\ &= 96 \text{ m} \end{aligned}$$

EXAMPLE 4

An automobile accelerates from 67.0 km/h to 96.0 km/h in 7.80 s. What is its acceleration (in m/s^2)?

Data:

$$\begin{aligned} v_f &= 96.0 \text{ km/h} \\ v_i &= 67.0 \text{ km/h} \\ t &= 7.80 \text{ s} \\ a &= ? \end{aligned}$$

Basic Equation:

$$a = \frac{v_f - v_i}{t}$$

Working Equation: Same

Substitution:

$$a = \frac{96.0 \text{ km/h} - 67.0 \text{ km/h}}{7.80 \text{ s}}$$

$$= \frac{29.0 \text{ km/h}}{7.80 \text{ s}}$$

$$= \frac{29.0 \frac{\cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1000 \text{ m}}{1 \cancel{\text{km}}} \times \frac{1 \cancel{\text{h}}}{3600 \text{ s}}}{7.80 \text{ s}}$$

$$= 1.03 \text{ m/s}^2$$

Freely falling bodies undergo constant acceleration. In a vacuum (in the absence of air resistance) a ball and a feather fall at the same rate. In 1971, astronaut David Scott demonstrated this during a moon mission by dropping a hammer and a feather simultaneously on the surface of the moon, where there is no atmosphere. Both objects fell at the same rate and landed at the same time.

TRY THIS ACTIVITY

Free Fall in a Vacuum

Drop a piece of paper and a book at the same time and note the relative time it takes for each to hit the floor. Now place that paper on top of the book as shown in Fig. 4.14. (***Note:*** The top surface area of the book must be larger than that of the paper.) What happens to the time it takes the book and paper to fall? What does this show about objects falling in a vacuum? (A vacuum is a space in which there is no air resistance present.)

Figure 4.14 Place the paper on top of the book. The book must be larger than the paper.

The acceleration of a freely falling body, also called the **acceleration due to gravity**, is denoted by the symbol g, where $a = g = 9.80 \text{ m/s}^2$ (metric system) or $a = g = 32.2 \text{ ft/s}^2$ (U.S. system) on the earth's surface.

What does $a = 9.80 \text{ m/s}^2$ mean? When a ball is dropped from a building, the speed of the ball increases by 9.80 m/s during each second. Figure 4.15 shows the total distance traveled and the speed at 1-s intervals.

We limit our discussion to uniformly accelerated motion because the mathematical tools needed to study other kinds of motion are beyond the scope of this book. We also need to assume that air resistance of a freely falling body is negligible for our calculations. However, air resistance is, in fact, an important factor in the design of machines that move through the atmosphere.

When the air resistance of a falling object equals its weight, the net force is zero and no further acceleration occurs. That is, a falling object reaches its **terminal speed**.

Figure 4.15 A ball falls with constant acceleration $a = g = 9.80\ \text{m/s}^2$ with the speed and the distance traveled calculated at the given times. Because the ball was dropped, $v_i = 0$ and the formulas for v_f and s are shown simplified. Note how the velocity and the distance traveled increase during each successive time interval.

Time	Distance Traveled	Speed
	$s = \frac{1}{2}at^2$	$v_f = at$
$t = 0$	0 m	0 m/s
$t = 1.00$ s	$s = \frac{1}{2}(9.80\ \text{m/s}^2)(1.00\ \text{s})^2$ $s = 4.90$ m	$v_f = (9.80\ \text{m/s}^2)(1.00\ \text{s})$ $v_f = 9.80$ m/s
$t = 2.00$ s	$s = \frac{1}{2}(9.80\ \text{m/s}^2)(2.00\ \text{s})^2$ $s = 19.6$ m	$v_f = (9.80\ \text{m/s}^2)(2.00\ \text{s})$ $v_f = 19.6$ m/s
$t = 3.00$ s	$s = \frac{1}{2}(9.80\ \text{m/s}^2)(3.00\ \text{s})^2$ $s = 44.1$ m	$v_f = (9.80\ \text{m/s}^2)(3.00\ \text{s})$ $v_f = 29.4$ m/s
$t = 4.00$ s	$s = \frac{1}{2}(9.80\ \text{m/s}^2)(4.00\ \text{s})^2$ $s = 78.4$ m	$v_f = (9.80\ \text{m/s}^2)(4.00\ \text{s})$ $v_f = 39.2$ m/s

TRY THIS ACTIVITY

Calculating Height

Drop a ball from a window that is at least two stories high (or from some other height) and time how long it takes the ball to fall to the ground. Use the formulas for accelerated motion to find the height. Measure the actual height with a tape measure. How does your calculated height compare with the actual height? What factors may have made your calculated height different from the actual height? Remember to use the problem-solving method to help you solve this problem.

EXAMPLE 5

A rock is thrown straight down from a cliff with an initial velocity of 10.0 ft/s. Its final velocity when it strikes the water below is $31\bar{0}$ ft/s. The acceleration due to gravity is $32.2\ \text{ft/s}^2$. How long is the rock in flight?

Data:

$$v_i = 10.0 \text{ ft/s}$$
$$a = 32.2 \text{ ft/s}^2$$
$$v_f = 31\bar{0} \text{ ft/s}$$
$$t = ?$$

Note the importance of listing all the data as an aid to finding the basic equation.

Basic Equation:

$$v_f = v_i + at \quad \text{or} \quad a = \frac{v_f - v_i}{t} \qquad \text{(two forms of the same equation)}$$

Working Equation:

$$t = \frac{v_f - v_i}{a}$$

Substitution:

$$t = \frac{31\bar{0} \text{ ft/s} - 10.0 \text{ ft/s}}{32.2 \text{ ft/s}^2}$$
$$= \frac{30\bar{0} \text{ ft/s}}{32.2 \text{ ft/s}^2}$$
$$= 9.32 \text{ s}$$

$$\boxed{\frac{\text{ft/s}}{\text{ft/s}^2} = \frac{\text{ft}}{\text{s}} \div \frac{\text{ft}}{\text{s}^2} = \frac{\text{ft}}{\text{s}} \cdot \frac{\text{s}^2}{\text{ft}} = \text{s}}$$

When any object is launched vertically upward, its upward speed is uniformly decreased by the force of gravity until it stops for an instant at its peak before falling back to the ground. As it is falling to the ground, it is uniformly accelerated by gravity the same as it would have been if dropped from its peak height. If an object is thrown vertically upward and if the initial velocity is known, the previous acceleration/gravity formulas may be used to find how high the object rises, how long it is in flight, and so on.

Note: When we consider a problem involving an object being thrown upward, we will consider an upward direction to be negative and the opposing gravity in its normal downward direction to be positive.

EXAMPLE 6

A baseball is thrown vertically upward with an initial velocity of 25.0 m/s (see Fig. 4.16). (a) How high does it go? (b) How long does it take to reach its maximum height? (c) How long is it in flight?

(a) Data:

$v_i = -25.0 \text{ m/s}$ (v_i is negative because the initial velocity is directed opposite gravity, g.)

$v_f = 0$ (At the instant of the ball's maximum height, its velocity is zero.)

$a = g = 9.80 \text{ m/s}^2$

$s = ?$

Basic Equation:

$$2as = v_f^2 - v_i^2$$

Working Equation:

$$s = \frac{v_f^2 - v_i^2}{2a}$$

Substitution:

$$s = \frac{0^2 - (-25.0\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)}$$

$$= -31.9\ \text{m}$$

$$\boxed{\frac{(\text{m/s})^2}{\text{m/s}^2} = \frac{\text{m}^2/\text{s}^2}{\text{m/s}^2} = \frac{\text{m}^2}{\text{s}^2} \div \frac{\text{m}}{\text{s}^2} = \frac{\cancel{\text{m}^2}^{\text{m}}}{\cancel{\text{s}^2}} \times \frac{\cancel{\text{s}^2}}{\cancel{\text{m}}} = \text{m}}$$

(s being negative indicates an upward displacement.)

(b) Data:

$$v_i = -25.0\ \text{m/s}$$
$$v_f = 0$$
$$a = g = 9.80\ \text{m/s}^2$$
$$t = ?$$

Basic Equation:

$$v_f = v_i + at$$

Working Equation:

$$t = \frac{v_f - v_i}{a}$$

Substitution:

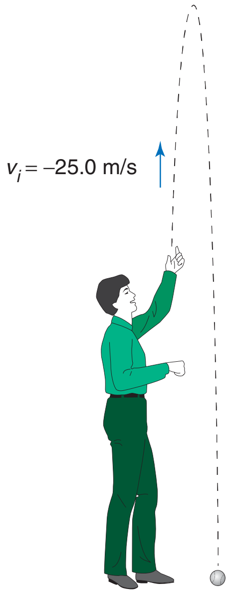
$$t = \frac{0 - (-25.0\ \text{m/s})}{9.80\ \text{m/s}^2}$$

$$= 2.55\ \text{s}$$

$$\boxed{\frac{\text{m/s}}{\text{m/s}^2} = \frac{\text{m}}{\text{s}} \div \frac{\text{m}}{\text{s}^2} = \frac{\cancel{\text{m}}}{\cancel{\text{s}}} \times \frac{\cancel{\text{s}^2}^{\text{s}}}{\cancel{\text{m}}} = \text{s}}$$

(c) The ball decelerates on the way up and accelerates on the way down at the same rate because the acceleration due to gravity is constant ($9.80\ \text{m/s}^2$). Therefore, the time for the ball to reach its peak is the same as the time for it to fall to the ground. The total time in flight is $2(2.55\ \text{s}) = 5.10\ \text{s}$.

Figure 4.16



With what speed does the ball in Example 6 hit the ground? The answer is 25.0 m/s. Can you explain why?

Earlier, we assumed no air resistance. We know that two dense and compact objects, such a bowling ball and a marble, will fall at the same rate in air. We also know that two unlike objects, such as a marble and a feather, fall at different rates because of air resistance.

A parachute takes advantage of air resistance to slow a sky diver's descent [Fig. 4.17(a)]. What would happen if a parachute does not open [Fig. 4.17(b)]? Would the sky diver's velocity increase constantly until he or she hits the ground? As the velocity increases, the air resistance also increases. Since the gravitational pull and the air resistance are directed opposite each other, they tend to oppose or equalize each other. (Here, the velocity and acceleration are both directed downward, while the air resistance is directed upward.) This equalization occurs when the friction of the air's resistance equals the force of gravity. When this equalization occurs, the falling object stops accelerating and continues to fall at a constant velocity, called *terminal velocity.*

The terminal velocity or speed of a sky diver varies from 150 to 200 km/h, depending on the person's weight and position. A heavier person will attain a greater terminal speed than a lighter person because the larger weight results in a larger acceleration before the air resistance equals the weight. Body position also makes a difference. When a body is spread out like that

Figure 4.17 The sky diver in (a) has a different acceleration toward the ground than those in (b) as a result of different air resistance.

(a) Courtesy of Photolibrary/Peter Arnold, Inc. (b) Courtesy of Getty Images, Inc. - Photodisc/Royalty Free

of a bird gliding with outstretched wings, its surface area increases, which results in more air resistance. Terminal speed can be controlled by varying the body position. A light sky diver and a heavy sky diver can remain in close proximity to each other if the light person decreases his or her air resistance by falling head or feet first while the heavy person spreads out and increases his or her air resistance. A parachute greatly increases air resistance and reduces the terminal speed to approximately 15 to 25 km/h, which is slow enough for a safe landing.

In general, the terminal velocity of an object varies with its weight and its aerodynamic features, which include the following:

1. The shape of the object. (A symmetrical object is more aerodynamic than a nonsymmetrical one.)
2. The orientation of the object as it is traveling. (A sky diver slows the fall by spreading out his or her arms and legs parallel to the ground while falling. The speed of the fall increases if the person falls head or feet first.)
3. The smoothness of the surface. (A body with a smooth surface provides less air resistance than a body with a rough surface and falls or flies faster as a result.)

TRY THIS ACTIVITY

Air Resistance and Acceleration

Safely drop several objects of varying sizes, weights, and shapes from a second- or third-floor window or balcony. Which objects reach the ground faster than others?

Gravity acts equally on all objects, but air resistance may cause some objects to accelerate less than 9.80 m/s^2. What aerodynamic features may have caused some objects to encounter more air resistance than others?

Finally, drop a coffee filter and observe the filter's descent to the ground (Fig. 4.18). Does it seem to accelerate as it falls? Why or why not?

Figure 4.18 Falling coffee filter

PROBLEMS 4.3

Substitute in the given equation and find the unknown quantity.

1. Given: $v_{avg} = \dfrac{v_f + v_i}{2}$
   $v_f = 6.20$ m/s
   $v_i = 3.90$ m/s
   $v_{avg} = ?$

2. Given: $a = \dfrac{v_f - v_i}{t}$
   $a = 3.07$ m/s^2
   $v_f = 16.8$ m/s
   $t = 4.10$ s
   $v_i = ?$

3. Given: $s = v_i t + \frac{1}{2}at^2$
   $t = 3.00$ s
   $a = 6.40$ m/s^2
   $v_i = 33.0$ m/s
   $s = ?$

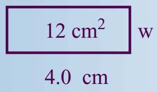
4. Given: $2as = v_f^2 - v_i^2$
   $a = 8.41$ m/s^2
   $s = 4.81$ m
   $v_i = 1.24$ m/s
   $v_f = ?$

5. Given: $v_f = v_i + at$
   $v_f = 10.40$ ft/s
   $v_i = 4.01$ ft/s
   $t = 3.00$ s
   $a = ?$

6. The average velocity of a mini-bike is 15.0 km/h. How long does it take for the bike to go 35.0 m?
7. A sprinter starting from rest reaches a final velocity of 18.0 mi/h. What is her average velocity?
8. A coin is dropped with no initial velocity. Its final velocity when it strikes the earth is 50.0 ft/s. The acceleration due to gravity is 32.2 ft/s^2. How long does it take to strike the earth?
9. A front endloader accelerates from rest to 1.75 m/s in 2.50 s. How far does it travel in that time?
10. A mechanic test driving a car that she has just given a tune-up accelerates from rest to 50.0 m/s in 9.80 s. How far does she travel in that time?
11. A rocket lifting off from earth has an average acceleration of 44.0 ft/s^2. Its initial velocity is zero. How far into the atmosphere does it travel during the first 5.00 s, assuming that it goes straight up?
12. The final velocity of a truck is 74.0 ft/s. If it accelerates at a rate of 2.00 ft/s^2 from an initial velocity of 5.00 ft/s, how long will it take for it to attain its final velocity?
13. A truck accelerates from 85 km/h to $12\bar{0}$ km/h in 9.2 s. Find its acceleration in m/s^2.
14. How long does it take a rock to drop 95.0 m from rest? Find the final speed of the rock.
15. An aircraft with a landing speed of 295 km/h lands on an aircraft carrier with a landing area 205 m long. Find the minimum constant deceleration required for a safe landing.
16. A ball is thrown downward from the top of a 43.0-ft building with an initial speed of 62.0 ft/s. Find its final speed as it strikes the ground.
17. A car is traveling at $7\bar{0}$ km/h. It then uniformly decelerates to a complete stop in 12 s. Find its acceleration (in m/s^2).
18. A car is traveling at $6\bar{0}$ km/h. It then accelerates at 3.6 m/s^2 to $9\bar{0}$ km/h. (a) How long does it take to reach the new speed? (b) How far does it travel while accelerating?
19. A rock is dropped from a bridge to the water below. It takes 2.40 s for the rock to hit the water. (a) Find the speed (in m/s) of the rock as it hits the water. (b) How high (in metres) is the bridge above the water?
20. A bullet is fired vertically upward from a gun and reaches a height of $70\bar{0}0$ ft. (a) Find its initial velocity. (b) How long does it take to reach its maximum height? (c) How long is it in flight?

PROBLEM SOLVING

SKETCH



DATA

A = 12 cm^2, l = 4.0 cm, w = ?

BASIC EQUATION

A = lw

WORKING EQUATION

$$w = \frac{A}{l}$$

SUBSTITUTION

$$w = \frac{12\ \text{cm}^2}{4.0\ \text{cm}} = 3.0\ \text{cm}$$

21. A bullet is fired vertically upward from a gun with an initial velocity of $25\bar{0}$ m/s. (a) How high does it go? (b) How long does it take to reach its maximum height? (c) How long is it in flight?
22. A rock is thrown down with an initial speed of 30.0 ft/s from a bridge to the water below. It takes 3.50 s for the rock to hit the water. (a) Find the speed (in ft/s) of the rock as it hits the water. (b) How high is the bridge above the water?
23. John stands at the edge of a deck that is 25.0 m above the ground and throws a rock straight up with an initial speed of 10.0 m/s. (a) How long does it take to reach its maximum height? (b) What maximum height above the deck does it reach? (c) Assuming it misses the deck on its way down, at what speed does it hit the ground? (d) What total length of time is the rock in the air?
24. John stands at the edge of a deck that is 40.0 m above the ground and throws a rock straight up that reaches a height of 15.0 m above the deck. (a) What is the initial speed of the rock? (b) How long does it take to reach its maximum height? (c) Assuming it misses the deck on its way down, at what speed does it hit the ground? (d) What total length of time is the rock in the air?
25. John is standing on a steel beam 255.0 ft above the ground. Linda is standing 30.0 ft directly above John. (a) For John to throw a hammer up to Linda, at what initial speed must John throw the hammer for it to just reach Linda? (b) Suppose the hammer reaches the correct height, but Linda just misses catching it. How long does someone on the ground have to move out of the way from the time the hammer reaches its maximum height? (c) At what speed does the hammer hit the ground?
26. Kurt is standing on a steel beam 275.0 ft above the ground and throws a hammer straight up at an initial speed of 40.0 ft/s. At the instant he releases the hammer, he also drops a wrench from his pocket. Assume that neither the hammer nor the wrench hits anything while in flight. (a) Find the time difference between when the wrench and the hammer hit the ground. (b) Find the speed at which the wrench hits the ground. (c) Find the speed at which the hammer hits the ground. (d) How long does it take for the hammer to reach its maximum height? (e) How high above the ground is the wrench at the time the hammer reaches its maximum height?
27. One ball is dropped from a cliff. A second ball is thrown down 1.00 s later with an initial speed of 40.0 ft/s. How long after the second ball is thrown will the second ball overtake the first?
28. A car with velocity 2.00 m/s at $t = 0$ accelerates at 4.00 m/s^2 for 2.50 s. What is its velocity at $t = 2.50$ s?
29. A truck moving at 30.0 km/h accelerates at a constant rate of 3.50 m/s^2 for 6.80 s. Find its final velocity in km/h.
30. A bus accelerates from rest at a constant 5.50 m/s^2. How long will it take to reach 28.0 m/s?
31. A motorcycle slows from 22.0 m/s to 3.00 m/s with constant acceleration -2.10 m/s^2. How much time is required to slow down?

4.4 Projectile Motion

A **projectile** is a launched object that travels through the air but has no capacity to propel itself. Up to this point, we have discussed projectiles either being thrown straight up or dropped straight down. We now discuss objects being thrown at an angle. This type of motion uses the same principles and formulas as in the previous section on uniformly accelerated motion. However, two dimensions are needed to analyze projectile motion; x will represent horizontal and y will represent vertical.

A baseball pop fly to right field, a car driving off a cliff, and water flowing out of a hose are all examples of projectile motion. **Projectile motion** is the movement of a projectile as it travels through the air influenced only by its initial velocity and gravitational acceleration.

Consider the example of a ball being rolled across a table at a constant 2.00 m/s. The ball moves 2.00 m for every second that it travels. Assuming no friction, the ball would continue to roll at that same speed (Fig. 4.19). As the ball approaches and rolls off the edge of the table, the ball continues to move horizontally at 2.00 m/s until it strikes the floor. Horizontally, there is nothing causing the ball to speed up or slow down, so it continues with that same horizontal motion.

Figure 4.19 A ball rolling across a table with a constant velocity

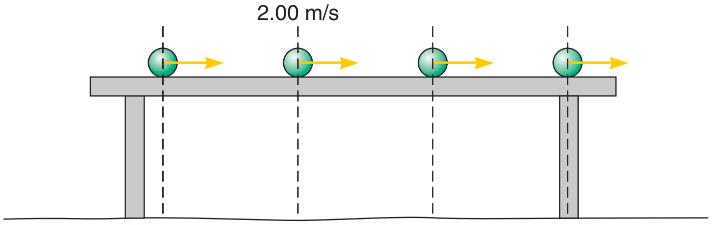
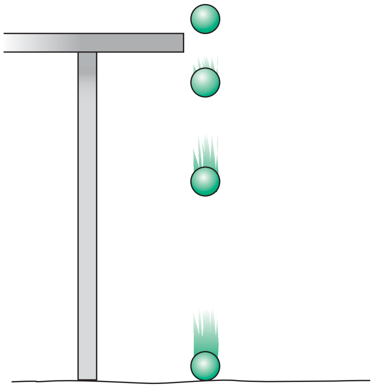


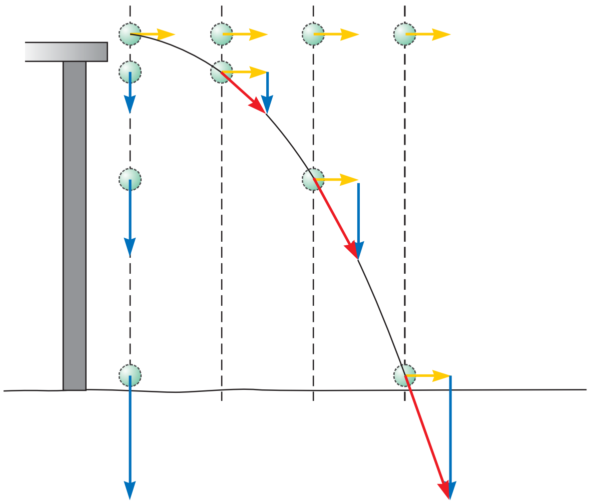
Figure 4.20 If a ball is simply dropped from the edge of a table, it will accelerate toward the ground, picking up speed and covering a greater and greater distance as time passes.



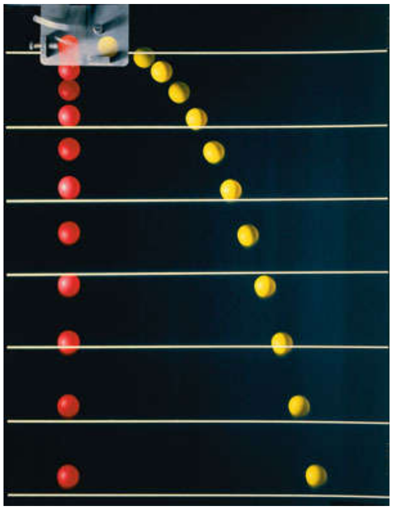
As the ball rolls off the table, gravity is accelerating the ball downward at a rate of $g = 9.80 \text{ m/s}^2$ (Fig. 4.20). As a result, the ball increases its vertical speed by 9.80 m/s for every second it falls. Vertically, the problem can be treated as any other uniformly accelerated motion problem, as seen in the previous section.

A projectile motion problem may be separated into the horizontal frame, the non-gravitational acceleration component, and into the vertical frame, the gravitational acceleration component. Using formulas in both the horizontal and vertical frames allows you to solve problems such as finding the speed the ball was traveling as it came off the table, the distance the ball landed from the edge of the table, and the time the ball was in the air (Fig. 4.21).

Figure 4.21



(a) This schematic shows the positions of a ball being dropped and a ball being projected horizontally from the same height at the same time. Note that the blue vertical component of the velocity of both balls is increasing and is the same for both balls at each instant, that the yellow horizontal component of the projected ball is constant at each instant, and that the incremental red resultant velocity of the projected ball is the resultant of each corresponding set of vertical and horizontal components, which results in the curved motion shown.



(b) Here a series of times flash photos shows that a dropped red ball and a yellow ball projected horizontally from the same height at the same time fall vertically at the same rate, and so they hit the ground at the same time, as demonstrated in the schematic in part (a).

TRY THIS ACTIVITY

Quarter and the Ruler Trick

Set up two coins and a ruler as shown in Fig. 4.22. Flick the ruler so the two coins are launched off the table at the same time. The coin closer to your stationary hand will travel more slowly than the coin farther away from your stationary hand. Which coin hits the ground first? Why?

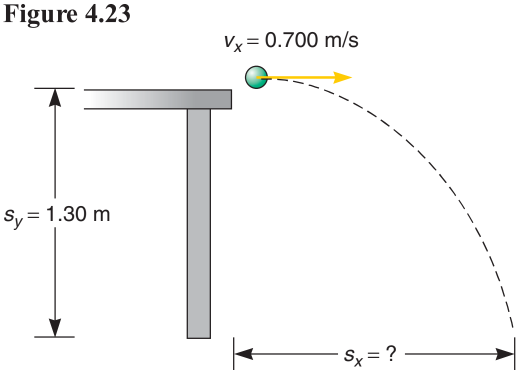
Figure 4.22

EXAMPLE 1

A ball rolls at a constant speed of 0.700 m/s as it reaches the end of a 1.30-m-high table (Fig. 4.23). How far from the edge of the table does the ball land?

Sketch:

Figure 4.23



Data:

$$v_{iy} = 0 \text{ m/s} \qquad v_x = 0.700 \text{ m/s}$$
$$s_y = 1.30 \text{ m} \qquad s_x = ?$$

Basic Equations:

$$s_y = v_{iy}\,t + \tfrac{1}{2}a_y t^2 \qquad s_x = v_x t$$

Working Equations (with $v_{iy} = 0$):

$$t = \sqrt{\frac{2s_y}{a}} \qquad s_x = v_x t$$

Substitution:

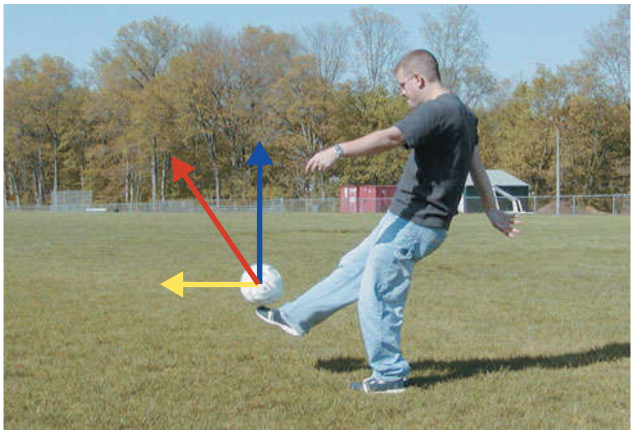
$$t = \sqrt{\frac{2(1.30 \text{ m})}{9.80 \text{ m/s}^2}}$$
$$= 0.515 \text{ s}$$

$$s_x = (0.700 \text{ m/s})(0.515 \text{ s})$$
$$= 0.361 \text{ m}$$

Projectiles launched at an angle need to be treated the same as a ball rolled off a table. However, unlike such a ball, which has an initial vertical velocity of zero, a projectile launched at an angle has an initial vertical velocity. The **range** is the horizontal distance that a projectile will travel before striking the ground. The range and the flight time may be found as follows.

1. Separate the original speed of the projectile into *horizontal* (x-component of the velocity) and *vertical components* (y-component of the velocity) using vectors and trigonometry (Fig. 4.24).

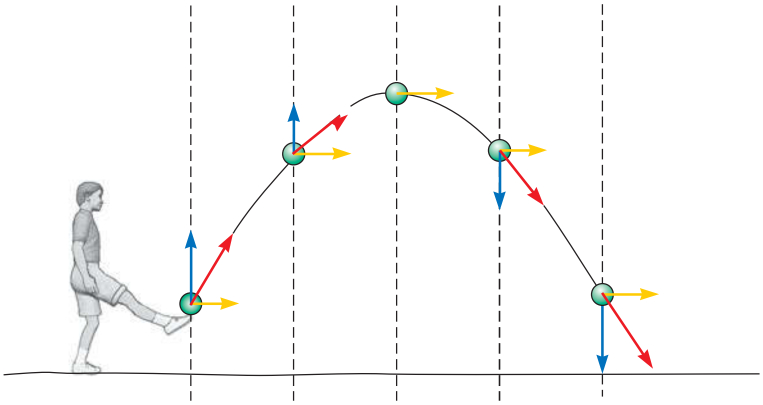
Figure 4.24 The ball is kicked with a horizontal (yellow vector) and vertical (blue vector) velocity. The red vector represents the resultant of the horizontal and vertical velocities.



2. Use the equation $s_x = v_x t$ to find the range. The horizontal component of the velocity does not change because gravitational acceleration does not act in the horizontal direction (Fig. 4.25). (Gravity pulls objects down in only the vertical direction.)
3. Use the vertical component of the velocity and uniformly accelerated motion equations to determine how long the projectile will be in the air. Since gravity accelerates the projectile, use the following formula and solve for time: $v_f = v_i + at$.

Note: Since the vertical components of v_i and v_f are equal but opposite in direction and therefore have opposite signs, the time for the projectile going up is the same as the time for it going down.

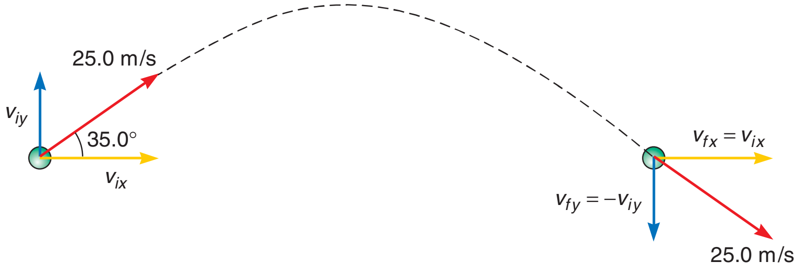
Figure 4.25 Velocity vectors are shown as a ball kicked into the air travels through its trajectory. Note that the yellow horizontal component remains constant at each instant while the blue vertical component decreases as the ball rises until the ball reaches its peak, at which the vertical component is zero, and then the vertical component increases as the ball falls.



EXAMPLE 2

A baseball is hit and moves initially at an angle of 35.0° above the horizontal ground with a velocity of 25.0 m/s as shown in Fig. 4.26. (a) What are the vertical and horizontal components of the initial velocity of the ball? (b) How long will the ball be in the air? (c) What will be the range for this projectile?

Figure 4.26



Data:

$$v = 25.0 \text{ m/s}$$
$$A = 35.0°$$

Basic Equations:

$$|\mathbf{v}_x| = |\mathbf{v}| \cos A$$
$$|\mathbf{v}_y| = |\mathbf{v}| \sin A$$
$$v_{fy} = v_{iy} + a_y t$$
$$s_x = v_x t$$

Working Equations: Same except for

$$t = \frac{v_{fy} - v_{iy}}{a_y}$$

Substitutions:

(a)
$$|\mathbf{v}_x| = (25.0 \text{ m/s})(\cos 35.0°)$$
$$= 20.5 \text{ m/s}$$
$$|\mathbf{v}_y| = (25.0 \text{ m/s})(\sin 35.0°)$$
$$= 14.3 \text{ m/s}$$

(b)
$$t = \frac{(14.3 \text{ m/s}) - (-14.3 \text{ m/s})}{9.80 \text{ m/s}^2}$$
$$= 2.92 \text{ s}$$

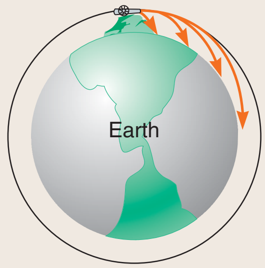
(c)
$$s_x = (20.5 \text{ m/s})(2.92 \text{ s})$$
$$= 59.9 \text{ m}$$

PHYSICS CONNECTIONS

Orbiting Cannonballs?

Isaac Newton once said that if he could fire a cannonball with enough velocity, he could get it to circle the globe. Is this true? Can you really put something into orbit by launching it fast enough? In reality, there is too much air resistance and too many obstacles to allow this to happen. Theoretically, though, if we launch something with enough horizontal velocity, the earth itself would curve away before the cannonball strikes it. Figure 4.27 shows a cannonball launched with varying initial velocities. As the horizontal velocity of the cannonball increases, its range increases as well. In addition, the ball appears to fall farther due to the curvature of the earth. Finally, when the ball is launched with a large enough velocity, it completely misses the earth and achieves orbit.

Figure 4.27 Newton's diagram of a cannonball orbiting the earth



To place a satellite into orbit, a rocket or space shuttle must bring the satellite to a point above the earth's atmosphere where it will not experience any air resistance. The object is then given enough horizontal velocity so that, although it is falling toward the earth, its horizontal velocity will prevent it from getting any closer to the surface. The concept of a falling object continuously missing the earth was a revolutionary concept developed by Isaac Newton over 300 years ago.

Communications satellites, the moon, and the astronauts in a space shuttle all are in a constant state of free fall but have the correct amount of horizontal velocity to prevent them from striking the earth. The speed required to keep a cannonball in orbit around the earth is approximately 17,700 mi/h. A space shuttle travels at approximately 17,400 mi/h because it orbits farther away from the earth.

PROBLEMS 4.4

Find the horizontal range for projectiles with the following speeds and angles.

	Angle	*Initial Speed*
1.	15.0°	35.0 m/s
2.	75.0°	35.0 m/s
3.	35.0°	35.0 m/s
4.	55.0°	35.0 m/s
5.	45.0°	35.0 m/s

6. Draw a conclusion about range and angles based on the answers to Problems 1 through 5.
7. Part of military training involves aiming and shooting a cannon. If a soldier sets the cannon at an angle of 62.0° and a launch velocity of 67.0 m/s, how far will the projectile travel horizontally?
8. A faulty fireworks rocket launches but never discharges. If the rocket launches with an initial velocity of 33.0 ft/s at an angle of 85.0°, how far away from the launch site does the rocket land?
9. An outfielder throws a baseball at a speed of $1\bar{1}0$ ft/s at an angle of 25.0° above the horizontal to home plate $2\bar{9}0$ ft away. Will the ball reach the catcher on the fly or will it bounce first?
10. A bearing rolls off a 1.40-m-high workbench with an initial horizontal speed of 0.600 m/s. How far from the edge of the bench does the bearing land?
11. A mechanic's socket rolls off a 1.50-m-high bench with an initial horizontal speed of 0.800 m/s. How far from the edge of the bench does the socket hit the floor?

Glossary

Acceleration Change in velocity per unit time. (p. 103)

Acceleration Due to Gravity The acceleration of a freely falling object. On the earth's surface the acceleration due to gravity is 9.80 m/s^2 (metric) or 32.2 ft/s^2 (U.S.) (p. 110)

Deceleration An acceleration that indicates an object is slowing down. (p. 106)

Motion A change of position. (p. 96)

Projectile A launched object that travels through the air but has no capacity to propel itself. (p. 116)

Projectile Motion The motion of a projectile as it travels through the air influenced only by its initial velocity and gravitational acceleration. (p. 117)

Range The horizontal distance that a projectile will travel before striking the ground. (p. 119)

Speed The distance traveled per unit of time. A scalar because it is described by a number and a unit, not a direction. (p. 96)

Terminal Speed The speed attained by a freely falling body when the air resistance equals its weight and no further acceleration occurs. (p. 110)

Velocity The rate of motion in a particular direction. The time rate of change of an object's displacement. Velocity is a vector that gives the direction of travel and the distance traveled per unit of time. (p. 96)

Formulas

4.1 $s = v_{avg}t$

4.2 $\Delta v = at$

4.3 $v_{avg} = \dfrac{v_f + v_i}{2}$ $\qquad s = v_it + \frac{1}{2}at^2$

$a = \dfrac{v_f - v_i}{t}$ $\qquad s = \frac{1}{2}(v_f + v_i)t$

$v_f = v_i + at$ $\qquad 2as = v_f^2 - v_i^2$

Review Questions

1. Velocity is
   (a) the distance traveled per unit of time.
   (b) the same as speed.
   (c) direction of travel and distance traveled per unit of time.
   (d) only the direction of travel.
2. A large heavy rock and a small marble are dropped at the same time from the roof of a three-story building. Neglecting air resistance, which object will strike the ground first?
   (a) The marble. (b) The rock.
   (c) They both strike the ground at the same time.
3. One ball is thrown horizontally while another is dropped vertically. Which ball will strike the ground first?
   (a) Both strike the ground at the same time.
   (b) The horizontally thrown ball strikes first.
   (c) The vertically dropped ball strikes first.
4. At what launch angle with the ground does a projectile have the greatest horizontal range?
   (a) 0° (b) 45° (c) 60° (d) 90°

5. Where in a projectile's path would its speed be the least?
   (a) Just as it is launched. (b) Just before it lands.
   (c) It has the same speed throughout its entire motion. (d) At the top of its path.
6. Explain your answer to Question 2.
7. Explain your answer to Question 3.
8. Distinguish between velocity and speed.
9. Is velocity always constant?
10. Why are vectors important in measuring motion? Provide two examples where vectors are used to help measure motion.
11. Give three familiar examples of acceleration.
12. Distinguish among acceleration, deceleration, and average acceleration.
13. State the values of the acceleration due to gravity for freely falling bodies in both the metric and U.S. systems.

Review Problems

1. A boat travels at 17.0 mi/h for 1.50 h. How far does the boat travel?
2. A commercial jet flies at $55\bar{0}$ mi/h for $30\bar{0}0$ mi. For how much time does the jet fly?
3. A plane flies north at 215 km/h. A wind from the east blows at 69 km/h. What is the plane's new velocity with respect to the ground in standard position?
4. A glider flies southeast (at 320.0°) at 25.0 km/h. A wind blows at 12.0 km/h from 15.0° south of west. What is the new velocity of the glider with respect to the ground in standard position?
5. A runner starts from rest and attains a speed of 8.00 ft/s after 2.00 s. What is the runner's acceleration?
6. A race car goes from rest to 150 km/h with an acceleration of 6.0 m/s^2. How many seconds does it take?
7. A sailboat has an initial velocity of 10.0 km/h and accelerates to 20.0 km/h. Find its average velocity.
8. A skateboarder starts from rest and accelerates at a rate of 1.30 m/s^2 for 3.00 s. What is his final velocity?
9. A plane has an average velocity of $5\bar{0}0$ km/h. How long does it take to travel 1.5×10^4 km?
10. A train has a final velocity of 110 km/h. It accelerated for 36 s at 0.50 m/s^2. What was its initial velocity?
11. A boulder is rolling down a hill at 8.00 m/s before it comes to rest 17.0 s later. What is its average velocity?
12. A truck accelerates from rest to 120 km/h in 13 s. Find its acceleration.
13. An airplane reaches a velocity of 71.0 m/s when it takes off. What must its acceleration be if the runway is 1.00 km long?
14. An airplane accelerates at 3.00 m/s^2 from a velocity of 21.0 m/s over a distance of 535 m. What is its final velocity?
15. A bullet is fired vertically upward and reaches a height of 2150 m. (a) Find its initial velocity. (b) How long does it take to reach its maximum height? (c) How long is it in flight?
16. A rock is thrown down with an initial speed of 10.0 m/s from a bridge to the water below. It takes 2.75 s for the rock to hit the water. (a) Find the speed of the rock as it hits the water. (b) How high is the bridge above the water?
17. A shot put is hurled at 9.43 m/s at an angle of 55.0°. Ignoring the height of the shot-putter, what is the range of the shot?
18. An archer needs to hit a bull's eye on a target at eye level 60.0 ft away. If the archer fires the arrow from eye level with a speed of 47.0 ft/s at an angle of 25.0° above the horizontal, will the arrow hit the target?

PROBLEM SOLVING

SKETCH

$\boxed{12 \text{ cm}^2}$ w

4.0 cm

DATA

$A = 12 \text{ cm}^2$, $l = 4.0$ cm, w = ?

BASIC EQUATION

$A = lw$

WORKING EQUATION

$$w = \frac{A}{l}$$

SUBSTITUTION

$$w = \frac{12 \text{ cm}^2}{4.0 \text{ cm}} = 3.0 \text{ cm}$$

APPLIED CONCEPTS

1. Amy walks at an average speed of 1.75 m/s toward her airport gate. When she comes within 57.5 m of her gate, she gets onto and continues to walk on a people mover at the same rate. (a) If she arrives at the gate 15.3 s after getting on the people mover, how fast was she moving relative to the ground? (b) What was the speed of the people mover?
2. A novice captain is pointing his ferryboat directly across the river at a speed of 15.7 mi/h. If he does not pay attention to the current that is headed downriver at 5.35 mi/h, what will be his resultant speed and direction? (Consider that the current and the boat's initial heading are perpendicular to each other.)
3. Anette is a civil engineer and needs to determine the length of a highway on-ramp before construction begins. If the average vehicle takes 10.8 s to go from 20.0 mi/h to 60.0 mi/h, how long should she design the separate merge lane to be so a car can reach a speed of 60.0 mi/h before merging?
4. As a movie stunt coordinator, you need to be sure a stunt will be safe before it is performed. If a stuntwoman is to run horizontally off the roof of a three-story building and land on a foam pad 7.35 m away from the base of the building, how fast should she run as she leaves the roof? (Each story is 4.00 m.)
5. As a newspaper delivery boy, Jason needs to know his projectile motion to throw a paper horizontally from a height of 1.40 m to a door that is 13.5 m away. What must the paper's velocity be for it to reach the door?